Control Systems Laboratory (EE 3321) — Experiment 4

Transfer Functions and Control System Characteristics

I. Overview of Experimental Procedure

This experiment utilizes the same theory as the previous two experiments to derive the transfer functions of both tanks in the Coupled Tank System. During this experiment, the student will investigate the dynamic behavior of Tanks 1 and 2. Since the open-loop system is stable, they will determine the required pump voltages to control the levels of Tanks 1 and 2 to desired levels in open loop.

II. Plant Description

In this experiment, we will analyze the Coupled-Tank plant. The hardware for this plant is available in the lab, allowing you to examine the actual system you will be working with. This plant will be used for hardware implementation in the final two experiments. Here, we will conduct software simulations for this plant.

A schematic of the Coupled-Tank plant is represented in Figure 2.1, below. As illustrated in Figure 2.1, the positive direction of vertical level displacement is upwards, with the origin at the bottom of each tank (i.e. corresponding to an empty tank).

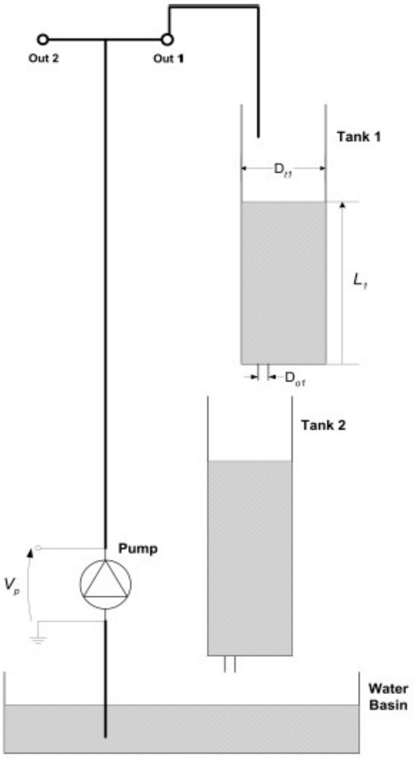


Figure 2.1: Schematic of Coupled Tank.

The system's two water tanks are made of Plexiglas tubes of uniform cross section. The Coupled-Tank pump is a gear pump composed of a DC motor rated for 12 V continuous and 22 V peak with heat radiating fins.

Each tank's actual liquid level is measured through a pressure sensor. Such a level sensor is located at the bottom of each tank and provides linear level readings over the complete liquid vertical level. In other words, the sensor output voltage increases proportionally to the applied pressure. Its output measurement is processed through a signal conditioning board and made available as 0 to 5V DC signal. Moreover, calibration of each pressure sensor's offset and gain potentiometers is required to keep level measurements consistent with the type of liquid used in the coupled-tank experiment.

III. Nonlinear Equation of Motion (EOM)

To derive the mathematical model of your Coupled-Tank system in Figure 2.1, it is reminded that the pump feeds into Tank 1 and that Tank 2 is not considered at all. Therefore, the input to the process is the voltage to the pump V_p and its output is the water level in tank 1, L_1, (i.e. top tank). The obtained Equation of Motion, EOM, should be a function of the system's input and output, as previously defined.

Therefore, you should express the resulting EOM under the following format:

$$\frac{\partial L_1}{\partial t} = f\left(L_1, V_p\right) \tag{2.1}$$

Where f denotes a function.

In deriving the Tank 1 EOM the mass balance principle can be applied to the water level in tank 1, i.e..

$$A_{t1}\frac{\partial L_1}{\partial t} = F_{i1} - F_{o1} \tag{2.2}$$

Where A_{t1} is the area of Tank1. F_{i1} and F_{o1} are the inflow rate and outflow rate, respectively. The volumetric inflow rate to tank 1 is assumed to be directly proportional to the applied pump voltage, such that:

$$F_{i1} = K_p V_p \tag{2.3}$$

Applying Bernoulli's equation for small orifices, the outflow velocity from tank 1, v_{o1}, can be expressed by the following relationship:

$$v_{o1} = \sqrt{2gL_1} \tag{2.4}$$

IV. EOM Linearization

Real systems often exhibit nonlinearity. However, to analyze system behavior or design a controller, we often work with linear systems. In control theory, we use Taylor series expansion to linearize a nonlinear system around an equilibrium point. This equilibrium point is where we want our system to settle in steady-state. Therefore, we can find the equilibrium point by setting all the derivatives of the nonlinear differential equation to zero.

The nonlinear EOM of tank 1 should be linearized around a quiescent point of operation. By definition, static equilibrium at a nominal operating point $\left(V_{p0}, L_{10}\right)$ is characterized by the Tank 1 level being at a constant position L_{10} due to a constant water flow generated by constant pump voltage V_{p0}.

In the case of the water level in tank 1, the operating range corresponds to small departure heights, L_{11}, and small departure voltages, V_{p1}, from the desired equilibrium point $\left(V_{p0}, L_{10}\right)$. Therefore, L_1 and V_p can be expressed as the sum of two quantities, as shown below:

$$L_1 = L_{10} + L_{11}, \;\; V_p = V_{p0} + V_{p1} \tag{2.5}$$

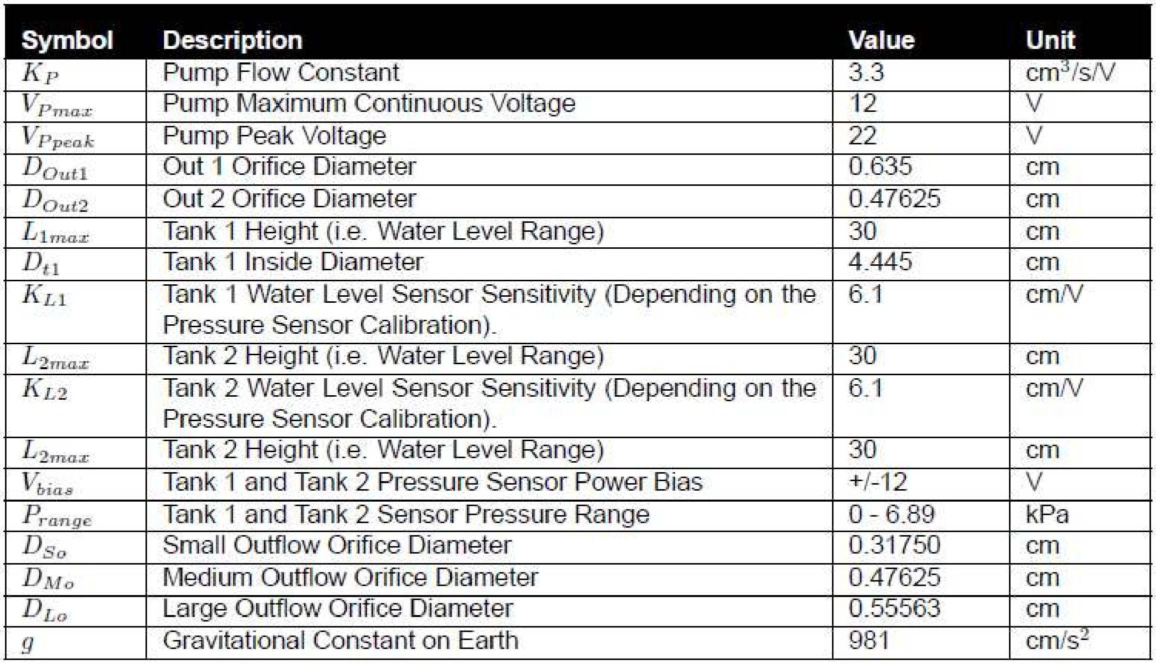
The obtained linearized EOM should be a function of the system's small deviations about its equilibrium point $\left(V_{p0}, L_{10}\right)$. Therefore, one should express the resulting linear EOM under the following format:

$$\frac{\partial L_{11}}{\partial t} = f\left(L_{11}, V_{p1}\right) \tag{2.6}$$

Where f denotes a function.

V. Coupled Tank Model Parameters

Below table, lists and characterizes the main parameters associated with the two-tank specialty plant. Some of these parameters can be used for mathematical modelling of the Coupled-Tank system as well as to obtain the water level's EOM.



Symbol	Description	Value	Unit
K_P	Pump Flow Constant	3.3	$cm^3/s/V$
V_{Pmax}	Pump Maximum Continuous Voltage	12	V
V_{Ppeak}	Pump Peak Voltage	22	V
D_{Out1}	Out 1 Orifice Diameter	0.635	cm
D_{Out2}	Out 2 Orifice Diameter	0.47625	cm
L_{1max}	Tank 1 Height (i.e. Water Level Range)	30	cm
D_{t1}	Tank 1 Inside Diameter	4.445	cm
K_{L1}	Tank 1 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
K_{L2}	Tank 2 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
V_{bias}	Tank 1 and Tank 2 Pressure Sensor Power Bias	+/-12	V
P_{range}	Tank 1 and Tank 2 Sensor Pressure Range	0 - 6.89	kPa
D_{So}	Small Outflow Orifice Diameter	0.31750	cm
D_{Mo}	Medium Outflow Orifice Diameter	0.47625	cm
D_{Lo}	Large Outflow Orifice Diameter	0.55563	cm
g	Gravitational Constant on Earth	981	cm/s^2

VI. Experimental Procedure

1) Derive the Equation of Motion (EOM) characterizing the dynamics of tank 2. Is the tank 2 system's EOM linear?
2) Find the nominal level of the tank 1 at system's static equilibrium. By definition, static equilibrium at a nominal operating point (L_{10}, L_{20}) is characterized by the water in tank 2 being at a constant position level L_{20} due to the constant inflow rate generated by L_{10}. (Assume $L_{20} = 15\ cm$).
3) Linearize tank 2 water level's EOM found in Problem 1 about the quiescent operating point (L_{10}, L_{20}).
4) Derive the transfer of function of the tank 2. Consider the L_2 as the output and L_1 as the input.
5) Using the linearized equations in experiment 2, derive the transfer of function of the tank 1. Consider the L_1 as the output and V_p as the input of the tank 1.
6) Using the two transfer functions, obtain the transfer function of the entire cascade system. Consider the L_2 as the output and V_p as the input.
7) Verify the transfer function of the entire system in MATLAB Simulink.

   Hint: You need to simulate the nonlinear equations and compare with the transfer function.
8) Determine the settling time of the transfer function theoretically and verify it through simulation.
9) Consider the standard second-order transfer function equation:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Assume your desire to have a dynamic response with $\xi = 1$, and the $\omega_n = 4$. How we can design the coupled-tank systems to have the desired behavior. Is there any other way without violating the integrity of the already designed system?!